R0904

Sub. Code
511201

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Second Semester

Mathematics

LINEAR ALGEBRA

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing correct option.

1. If $S = \{v_1, v_2, ..., v_n\}$ is a set of vectors in a finite dimensional vector space V, then S is called a basis for V if: (CO1, K2)

   (a) S spans V (b) S is linearly independent

   (c) either (a) or (b) (d) both (a) and (b)

2. If B is a basis for a vector space R^n, then $P_{B \to B}$ is (CO1, K1)

   (a) the zero matrix

   (b) a diagonal matrix

   (c) no particular matrix

   (d) the identity matrix

3. Let $T : R^5 \to R^3$ be the linear transformation defined by $T(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_2, x_2 + x_3 + x_4, x_4 + x_5)$. Find the nullity of the standard matrix for T. (CO2, K1)

   (a) 5 (b) 3

   (c) 2 (d) 1

4. Let V, U, W be finite dimensional vector spaces over the same field F and let $T: V \to U$ and $S: U \to W$ be linear maps then, (CO2, K1)

(a) Rank ST cannot exceed rank S or rank T

(b) Rank ST must exceed rank S or rank T

(c) Nullity $(ST) >$ Nullity T

(d) Nullity $(ST) \leq$ Nullity T

5. If V is a vector space of all polynomials of degree at most 10 with coefficients from a field F, then the dual space V^* is a vector space over F of dimension. (CO3, K1)

(a) 10 (b) 12

(c) 9 (d) 11

6. Let V be the subspace of the space of polynomials $R[x]$, of degree at most n. Let $\{f_1, f_2, .., f_{n+1}\}$ be a basis of V. Then (CO3, K1)

(a) Some f_1 has degree 0

(b) The f_1 have distinct degrees

(c) Some f_1 has degree n

(d) The f_1 have the same degrees

7. The value of the determinant $\begin{vmatrix} 1 & x & y \\ 0 & \cos x & \sin y \\ 0 & \sin x & \cos y \end{vmatrix}$ is ———— (CO4, K2)

(a) $\sin(x+y)$ (b) $\sin(x-y)$

(c) $\cos(x+y)$ (d) More than one of the above

8. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is ———— (CO4, K2)

(a) 4 (b) –4

(c) ± 4 (d) 0

9. Let T be a linear operator on a vector space V. A subspace W is invariant under T is ———— (CO5, K2)

(a) $T(W) = \varphi$ (b) $T(W) = W$

(c) $T(W) \subset W$ (d) $T(W) \cup W = \{0\}$

10. Let $T : V \to V$ be a linear transformation on a vector space V such that every one dimensional subspace of V is invariant under T. Then ———— (CO5, K1)

(a) T must be onto

(b) T must be one to one

(c) There exists a scalar a such that $Tv = av$ for all v

(d) If $\{v_1, v_2, ..., v_k\}$ is a basis of V then there exists distinct scalar constants $a_1, a_2, ..., a_k$ such $Tv_i = a_i v_i$ that.

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Prove that a non-empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar c in F the vector $c\alpha + \beta$ is again in W. (CO1, K3)

Or

(b) Let V be a vector space over the field F then prove that the intersection of any collection of subspaces of V is a subspace of V. (CO1, K2)

12. (a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite-dimensional. Then prove that $rank(T) + nullity(T) = \dim V$. (CO2, K2)

Or

(b) If A is an $m \times n$ matrix with entries in the field F, then prove that $row\ rank(A) = column\ rank(A)$. (CO2, K2)

13. (a) Let F be a subfield of the complex numbers and let A be the following 2×2 matrix over F, $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ For each of the following polynomials f over F, compute $f(A)$. (CO3, K2)

(i) $f = x^2 - x + 2$

(ii) $f = x^2 - 1$

(iii) $f = x^2 - 5x + 7$

Or

(b) If f and g are independent polynomials over a field F and h is a non-zero polynomial over F, show that fh and gh are independent. (CO3, K3)

14. (a) Let K be a commutative ring with identity and let n be a positive integer. Show that there exists at least one determinant function on $K^{n\times n}$. (CO4, K2)

Or

(b) Prove that a linear combination of $n-$linear functions is $n-$linear. (CO4, K3)

15. (a) Let V be a finite-dimensional vector space and let W_1 be any subspace of V. Prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$. (CO5, K4)

Or

(b) Prove that if E is the projection on R along N, then prove that $(I - E)$ is the projection on N along R. (CO5, K3)

Part C $(5 \times 8 = 40)$

Answer **all** questions not more than 1000 words each.

16. (a) Prove that the only subspaces of R^1 are R^1 and the zero subspace. (CO1, K3)

Or

(b) Prove that a subspace of R^2 is R^2, or the zero subspace, or consists of all scalar multiples of some fixed vector in R^2. (CO1, K2)

17. (a) Find the range, rank, null space, and nullity for the zero transformation and the identity transformation on a finite-dimensional space V. (CO2, K2)

Or

(b) Let V be an $n-$dimensional vector space over the field F, and let W be an $m-$dimensional vector space over F. Then prove that the space $L(V,W)$ is finite-dimensional and has dimension mn. (CO2, K2)

18. (a) If F is a field, show that the product of two non-zero elements of F^∞ is non-zero. (CO3, K2)

Or

(b) Let f be a polynomial over the field F, and let c be an element of F. Then prove that f is divisible by $x - c$ if and only if $f(c) = 0$. (CO3, K3)

19. (a) Let K be a commutative ring with identity, and let D be an alternating 2-linear function on 2×2 matrices over K. Show that $D(A) = (\det A)D(I)$ for all A. Show that det $(AB)(\det A)(\det B)$ for any 2×2 matrices A and B over K. (CO4, K2)

Or

(b) Let F be a field and D a function on $n \times n$ matrices over F (with values in F). Suppose $D(AB) = D(A)D(B)$ for all A, B. Show that either $D(A) = 0$ for all A, or $D(I) = 1$. In the latter case show that $D(A) \neq 0$ whenever A is invertible. (CO4, K3)

20. (a) Let T be a linear operator on V. If every subspace of V is invariant under T, then prove that T is a scalar multiple of the identity operator. (CO5, K4)

Or

(b) Let T be a linear operator on an $n-$dimensional space, and suppose that T has n distinct characteristic values. Prove that any linear operator which commutes with T is a polynomial in T. (CO5, K3)

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R0905

Sub. Code
511202

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. The generalization of Riemann integral is called _________ . (CO1, K3)

   (a) Lebesgue integral

   (b) Riemann-Stieltjes integral

   (c) Bounded integral

   (d) R-S integral

2. In Riemann-Stieltjes integral, the function f is called _________. (CO1, K3)

   (a) integrator (b) integrand

   (c) integral (d) real function

3. A series $\sum a_n$ is called conditionally convergent if $\sum |a_n|$ _________ . (CO2, K3)

   (a) converges (b) weekly converges

   (c) diverges (d) strongly converges

4. In a Bernoulli's polynomial, $B_0 =$ _________. (CO2, K4)

(a) 2 (b) ½

(c) – ½ (d) 1

5. The sequence of function $\{f_n\}$ is point wise bounded on a set E, if $\{f_n(x)\}$ is _________ for all $x \in E$. (CO3, K3)

(a) bounded (b) unbounded

(c) continuous (d) discontinuous

6. If f is continuous on [0, 1], then the integral of the product of f with any polynomial is _________. (CO3, K4)

(a) 1 (b) 0

(c) 2 (d) 4

7. The value $E(\pi i) =$ _________. (CO4, K5)

(a) 1 (b) 2

(c) –1 (d) –2

8. The value $C\left(\frac{\pi}{2}\right) =$ _________. (CO4, K5)

(a) 1 (b) –1

(c) ± 1 (d) 0

9. Define $a_{ij} = \begin{cases} 0, \, if \;\, i < j \\ -1, \, if \;\, i = j \\ 2^{j-i}, \, if \;\, i > j \end{cases}$, the value of $\sum_j \sum_i a_{ij} =$ _______.

(CO5, K4)

(a) 0 (b) –2

(c) 3 (d) –1

10. The value of $\Gamma(6) =$ _________. (CO5, K5)

(a) 24 (b) 120

(c) 720 (d) 6

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) If f is continuous on $[a, b]$ and if α is of bounded variation on $[a, b]$, then show that $f \in R(\alpha)$ in $[a, b]$. (CO1, K2)

Or

(b) Change of variable in a Riemann integral – Explain. (CO1, K2)

12. (a) Cauchy condition for Uniform convergence – Explain. (CO2, K4)

Or

(b) Let $a_n = 2\sqrt{n} - \sum_{k=1}^{n} 1/\sqrt{k}$. Prove that the sequence $\{a_n\}$ converges to a limit p in the interval (1,2).

(CO2, K5)

13. (a) Suppose $\{f_n\}$ is a sequence of functions defined on E and suppose $|f_n(x)| \le M_n$, (where $x \in E$, $n = 1,2,3,\ldots$) If $\sum M_n$ converges, then show that $\sum f_n$ converges uniformly on E. (CO3, K2)

Or

(b) There exists a real continuous function on the real line is nowhere differentiable – Prove. (CO3, K4)

14. (a) Given a double sequence $\{a_{ij}\}$, $i = 1,2,3,\ldots,\ j = 1,2,3,\ldots$ Suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i$, $i = 1,2,3,\ldots$ and $\sum b_i$ converges. Then show that $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$. (CO4, K2)

Or

(b) Suppose $\sum c_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n (-1 < x < 1)$. Then prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$. (CO4, K4)

15. (a) If $0 < x < \infty$, then show that $\Gamma(x+1) = x\Gamma(x)$. (CO5, K2)

Or

(b) State and Prove Striling's formula. (CO5, K5)

Part C $(5 \times 8 = 40)$

Answer **all** questions not more than 1000 words each.

16. (a) State and Prove First Mean-Value theorem for Riemann integral. (CO1, K5)

Or

(b) Assume that α is of bounded variation on $[a, b]$. Let $V(x)$ denote the total variation of α on $[a, x]$ if $a < x \leq b$ and let $V(a) = 0$. Let f be defined and bounded on $[a, b]$. If $f \in R(\alpha)$ on $[a, b]$, then prove that $f \in R(V)$ on [a,b]. (CO1, K5)

17. (a) State and Prove Dirichlet's test for uniform convergence. (CO2, K5)

Or

(b) State and Prove Bernstein theorem. (CO2, K5)

18. (a) State and Prove Stone Weierstrass theorem. (CO3, K5)

Or

(b) If K is a compact metric space, if $f_n \in \wp(K)$ for $n = 1, 2, 3, \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that (i) $\{f_n\}$ is uniformly bounded on K, (ii) $\{f_n\}$ contains a uniformly convergent subsequence. (CO3, K4)

19. (a) State and Prove Taylor's theorem. (CO4, K5)

Or

(b) Prove that the function E is periodic, with period $2\pi i$. Prove that the functions C and S are periodic, with period 2π. (CO4, K5)

20. (a) State and Prove Parseval's theorem. (CO5, K5)

Or

(b) If f is positive function on $(0, \infty)$ such that $f(x+1) = xf(x)$, $f(1) = 1$, $\log f$ is convex, then prove that $f(x) = \Gamma(x)$. (CO5, K4)

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R0906

Sub. Code
511203

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Which of the following statements is true for an analytic function in complex analysis? (CO1, K1)

   (a) It must be differentiable at all points in the complex plane

   (b) It must be differentiable only on the real axis

   (c) It must be differentiable only on the imaginary axis

   (d) It cannot be differentiable anywhere in the complex plane

2. If $f(z) = e^{iz} + \sin(z)$, where z is a complex variable, what is the condition for $f(z)$ to be analytic? (CO1, K1)

   (a) e^{iz} is analytic, but sin z is not

   (b) $\sin z$ is analytic, but e^{iz} is not

   (c) both e^{iz} and sin z are analytic

   (d) none of them are analytic

3. What is the complex integral of $f(z)=z^2$ along the straight line segment from $z=1$ to $z=3$? (CO2, K1)

(a) $8+12i$ (b) $8-12i$

(c) $12+8i$ (d) $12-8i$

4. If $f(z)$ is analytic and on a circle C with center z_0 and radius R, what is the expression for $f(z_0)$ according to Cauchy's Integral formula? (CO2, K1)

(a) $f(z_0)=\frac{1}{2\pi i}\int_0^{2\pi}\frac{f(z)}{z-z_0}d\theta$

(b) $f(z_0)=\frac{1}{2\pi i}\int_0^{2\pi}\frac{f\left(z_0+\mathrm{Re}^{i\theta}\right)}{\mathrm{Re}^{i\theta}}d\theta$

(c) $f(z_0)=\frac{1}{2\pi}\int_0^{2\pi}\frac{f\left(z_0+\mathrm{Re}^{i\theta}\right)}{\mathrm{Re}^{i\theta}}d\theta$

(d) $f(z_0)=\frac{1}{\pi i}\int_0^{\pi}\frac{f(z)}{z-z_0}d\theta$

5. What is the definition of a zero of a complex function? (CO3, K1)

(a) The value where the function is undefined

(b) The value where the function is equal to zero

(c) The value where the function is infinite

(d) The value where the function has a jump discontinuity

6. What is the order of a pole at a point for a complex function? (CO3, K1)

(a) The number of zeroes at that point

(b) The number of times the function is undefined at that point

(c) The reciprocal of the number of zeroes at that point

(d) The reciprocal of the number of times the function is undefined at that point

7. For a function $f(z)$ with a simple pole at $z = 2i$, what is the residue of $f(z)$ at the pole? (CO4, K1)

(a) $2i$ (b) $-2i$

(c) 2 (d) –2

8. Which statement is true about the residue at a simple pole of a complex function? (CO4, K1)

(a) It is always zero (b) It is infinite

(c) It is undefined (d) It is finite and non-zero

9. What is the radius of convergence of a Taylor series? (CO5, K1)

(a) The distance to the nearest singularity of the function

(b) The distance to the farthest singularity of the function

(c) The distance to the point of expansion

(d) The distance to the point of expansion divided by the number of terms in the series

10. What is a Laurent series used for? (CO5, K1)

(a) Representing functions as power series centered at a point

(b) Representing functions as power series on a region containing a point and an annulus

(c) Representing function as trigonometric series

(d) Representing function as series involving factorial terms

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) If all zeros of a polynomial $P(z)$ lie in a half plane, then prove that all zeros of the derivative $p'(z)$ lie in the same half plane. (CO1, K2)

Or

(b) Derive the complex form of the Cauchy-Riemann equations. (CO1, K2)

12. (a) State and prove Cauchy's representation formula. (CO2, K5)

Or

(b) Suppose that $f(z)$ is analytic on a closed curve γ (i.e., f is analytic in a region that contains γ). Show that $\int_{\gamma} \overline{f(z)} f'(z) dz$ is purely imaginary. (The continuity of $f'(z)$ is taken for granted.) (CO2, K2)

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighbor hood of an essential singularity. (CO3, K5)

Or

(b) State and prove Taylor's theorem. (CO3, K5)

14. (a) Evaluate the following integrals by the method of residues (CO4, K6)

(i) $\int_0^\infty \frac{x^2 dx}{x^4 + 5x^2 + 6}$

(ii) $\int_0^\infty \frac{x^2 dx}{\left(x^2 + a^2\right)^3}$, a real.

Or

(b) State and prove maximum principle for harmonic function. (CO4, K5)

15. (a) Define an entire function with an example. Also Prove that every function which is meromorphic in the whole plane is the quotient of two entire functions. (CO5, K2)

Or

(b) If the functions $f_n(z)$ are analytic and $\neq 0$ in a region Ω, and if $f_n(z)$ converges to $f(z)$, uniformly on every compact subset of Ω, then prove $f(z)$ is either identically zero or never equal to zero in Ω. (CO5, K5)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) If $\sum a_n z^n$ and $\sum b_n z^n$ have radii of convergence R_1 and R_2, show that the radius of convergence of $\sum a_n b_n z^n$ is at least $R_1 R_2$. (CO1, K4)

Or

(b) Prove Cauchy's inequality by induction. (CO1, K5)

17. (a) State and prove the Cauchy's theorem for rectangle. (CO2, K5)

Or

(b) State and prove the local mapping theorem. (CO2, K5)

18. (a) State and prove Taylor's theorem. (CO3, K5)

Or

(b) If $f(z)$ is defined and continuous on a closed bounded set E and analytic on the interior of E, then prove the maximum of $|f(z)|$ on E is assumed on the boundary of E. (CO3, K6)

19. (a) State and prove arguments principle. (CO4, K5)

Or

(b) If the piecewise differentiable closed curve γ does not pass through the point a, then show the value of the integral $\int_{\gamma} \frac{dz}{z - a}$ is a multiple of $2\pi i$. (CO4, K2)

20. (a) Derive Laurent series. (CO5, K6)

Or

(b) Prove that the Laurent development is unique. (CO5, K5)

R0907

Sub. Code
511204

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. The primitive of the equation

   $\left(x^2 z - y^3\right) dx + 3xy^2 dy + x^3 dz = 0$ is (CO1, K1)

   (a) $x^2 z + y^3 = cx$ (b) $x^2 z + y^2 = cx$

   (c) $x^3 z + y^2 = cx$ (d) $x^3 z + y^3 = cx$

2. Solution of the equation

   $(y+z)\, dx + (z+x) dy + (x+y) dz = 0$ is (CO1, K2)

   (a) $x^2 y + y^2 z + z^2 x = C$

   (b) $xz + zy + yx = Cv$

   (c) $xy + yz + zx = C$

   (d) $x^2 z + z^2 y + y^2 x = C$

3. The particular integral of the equation $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = xy$ is (CO2, K1)

(a) $x + y \log x$ (b) $xy \log x$

(c) $\log y + y \log x$ (d) $y + x \log y$

4. The canonical form of the parabolic partial differential equation is? (CO2, K1)

(a) $\frac{\partial^2 z}{\partial v^2} = \varphi(u, v, z, z_u, z_v)$

(b) $\frac{\partial^2 z}{\partial u \partial v} = \varphi(u, v, z, z_u, z_v)$

(c) $\frac{\partial^2 z}{\partial v^2} = \varphi(0)$

(d) None of these

5. Charpit's equation for the given partial differential equation $z = pq$ is (CO3, K1)

(a) $\frac{dp}{p} = \frac{dq}{q} = \frac{dz}{-2pq} = \frac{dx}{-q} = \frac{dy}{-p}$

(b) $\frac{dp}{-p} = \frac{dq}{-q} = \frac{dz}{-2pq} = \frac{dx}{q} = \frac{dy}{p}$

(c) $\frac{dp}{-p} = \frac{dq}{-q} = \frac{dz}{-2pq} = \frac{dx}{-q} = \frac{dy}{-p}$

(d) $\frac{dp}{-p} = \frac{dq}{-q} = \frac{dz}{2pq} = \frac{dx}{-q} = \frac{dy}{-p}$

6. Which of the following equation satisfies $f(p,q)=0$? (CO3, K1)

(a) Charpit's Equation (b) Clairaut Equation

(c) Green's Equation (d) Separable Equation

7. The solution of the equation $ye^{xy}dx \mid (xe^{xy}+2y)\,dy=0$ is (CO4, K2)

(a) $xe^{xy}+2y^2=c$ (b) $xe^{xy}+y^2=c$

(c) $e^{xy}+2y^2=c$ (d) $e^{xy}+y^2=c$

8. The solution of the equation $\frac{dy}{dx}=y^2$ with initial value $y(0)=1$ is bounded in the interval (CO4, K2)

(a) $-\infty \le x \le \infty$ (b) $-\infty \le x \le 1$

(c) $x<1, x>1$ (d) $-2 \le x \le 2$

9. In the stead state of a two-dimensional heat flow, if u is independent of t, then the equation reduces to ——————. (CO5, K1)

(a) Heat flow in three dimensions

(b) Laplace in three dimensions

(c) Elliptic in two dimensions

(d) Heat flow in two dimensions

10. The suitable solution of one dimensional diffusion equation is (CO5, K1)

(a) $u = Ae^{\lambda x}+Be^{-\lambda x}Ce^{c^2\lambda^{2t}}$

(b) $u(A\cos\lambda x + B\sin\lambda x)Ce^{c^2\lambda^{2t}}$

(c) $u=(Ax+B)\,C$

(d) None of these

Section B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$. (CO1, K3)

Or

(b) State and prove the necessary and sufficient condition that the Pfaffian differential equation $X.dr = 0$ to be integrable. (CO1, K3)

12. (a) Find the integral surface of the linear partial differential equation

$x\left(y^2 + z\right) p - y\left(x^2 + z\right) q = \left(x^2 - y^2\right) z$ which contains the straight line $x + y = 0,\ z = 1$. (CO2, K5)

Or

(b) Verify that the equation $z = \sqrt{2x + a} + \sqrt{2y + b}$ is complete integral of the partial differential equation $z = \frac{1}{p} + \frac{1}{q}$. (CO2, K4)

13. (a) Find a complete integral of the equation $p^2 y\left(1 + x^2\right) = qx^2$. (CO3, K4)

Or

(b) Show that the equation $xpq + yq^2 = 1$ has complete integrals : (CO3, K3)

(i) $(z + b)^2 = 4(ax + y)$

(ii) $kx(z + h) = k^2 y + x^2$ and deduce(ii) from (i).

14. (a) If $u_1, u_2, ..., u_n$, are solutions of the homogeneous lienar partial differential equation $F(D, D')z = 0$, then show that $\sum_{r=1}^{n} c_r u_r$, where the c_r's are arbitrary constants, is also a solution. (CO4, K3)

Or

(b) Find the solution of one-dimensional diffusion equation $\frac{\partial^2 z}{\partial^2 x^2} = \frac{1}{k} \frac{\partial z}{k \partial t}$. (CO4, K3)

15. (a) If $\rho > 0$ and $\psi(r)$ is given by the equation $\psi(r) = \int_V \frac{\rho(r') d\tau'}{|r - r'|}$, where the volume V is bounded, prove that $\lim_{r=\infty} r\psi(r) = M$, where $M = \int_V \rho(r')\, d\tau'$. (CO5, K3)

Or

(b) Derive the elementary solution of Laplace's equation. (CO5, K3)

Section C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) Find the integral curves of the equations $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$. (CO1, K3)

Or

(b) State and prove the necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$, not involving x or y explicity. (CO1, K4)

17. (a) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x - axis. (CO2, K3)

Or

(b) Obtain the characteristic equation of the differential equation $F(x, y, z, p, q) = 0$. (CO2, K3)

18. (a) Find a complete integral of the equation $p^2 x + q^2 y = z$. (CO3, K2)

Or

(b) Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$. (CO3, K4)

19. (a) If $\beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\varepsilon)$ is an arbitrary function of the single variable ε, then show that if $\beta_r \neq 0$, $u_r = \exp\left(-\frac{\gamma_r y}{\beta_r}\right) \phi_r(\beta_r x)$ is a solution of the equation $F(D, D') = 0$. (CO4, K3)

Or

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it. (CO4, K3)

20. (a) Derive the elementary solution of the one-dimensional wave equation. (CO5, K3)

Or

(b) Obtain the solution of the finite string length l occupying the space $0 \leq x \leq l$. (CO5, K3)

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R0908

Sub. Code
511510

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Second Semester

Mathematics

Elective – INTRODUCTION TO PYTHON PROGRAMMING

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the objective questions by choosing the correct option.

1. What is the maximum possible length of an identifier? (CO1, K2)

 (a) 16 (b) 32

 (c) 64 (d) None of these above

2. What is the method inside the class in python language? (CO1, K1)

 (a) Object (b) Function

 (c) Attribute (d) Argument

3. Which of the following is not a keyword in Python language? (CO2, K1)

 (a) Val (b) raise

 (c) try (d) with

4. Which of the following operators is the correct option for power(ab)? (CO2, K1)

(a) a ^ b (b) a**b

(c) a ^^ b (d) a ^*b

5. The output to execute string.ascii_letters can also be obtained from: (CO3, K1)

(a) Character

(b) ascii_lowercase_string.digits

(c) lowercase_string.upercase

(d) ascii_lowercase+string.ascii_upercase

6. The set of statements that will be executed whether an exception is thrown or not? (CO3, K1)

(a) Except

(b) Else

(c) Finally

(d) Assert

7. The basic ndarray is created using? (CO4, K2)

(a) numpy.array(object, dtype = None, copy — True, subok = False, ndmin = 0)

(b) numpy.array(object, dtype = None, copy = True, order = None, subok = False, ndmin = 0)

(c) numpy_array(object, dtype = None, copy = True, order = None, subok = False, ndmin = 0)

(d) numpy.array(object, dtype = None, copy = True, order = None, ndmin = 0)

8. What is the name of the operator in Python?

(CO4, K2)

(a) Exponentiation

(b) Modulus

(c) Floor division

(d) None of the mentioned above

9. The % operator returns the ————. (CO5, K1)

(a) Quotient

(b) Divisor

(c) Remainder

(d) None of the mentioned above

10. Python supports the creation of anonymous functions at runtime, using a construct called ————. (CO5, K2)

(a) pi (b) anonymous

(c) lambda (d) beta

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Differentiate between Interpreter and Compiler. (CO1, K3)

Or

(b) Write a sort note on file management and basic tools in python. (CO1, K2)

12. (a) Write a program using functions to display Pascal's triangle. (CO2, K2)

Or

(b) Write a function called sum_digits that is given an integer num and returns the sum of the digits of num. (CO2, K2)

13. (a) Write a program that uses a while loop to add up all the even numbers between 100 and 200. (CO3, K2)

Or

(b) Write a function called factors that takes an integer and returns a list of its factors. (CO3, K3)

14. (a) Write a sort note on data types in Python. (CO4, K2)

Or

(b) Differentiate between NumPy and SciPy in Python. (CO4, K3)

15. (a) Write a program in Python to find GCD of two or more integers. (CO5, K4)

Or

(b) Write a program in Python to find prime numbers for the given_integers. (CO5, K3)

Part C $(5 \times 8 = 40)$

Answer **all** questions not more than 1000 words each.

16. (a) Explain precedence and associativity of operators with examples. (CO1, K3)

Or

(b) Briefly explain binary left shift and binary right shift operators with examples. (CO1, K2)

17. (a) Write a function called **is_sorted** that is given a list and returns True if the list is sorted and False otherwise. (CO2, K2)

Or

(b) Write a function called root that is given a number x and an integer n and returns $x^{\frac{1}{n}}$. In the function definition, set the default value of n to 2. (CO2, K2)

18. (a) Write a program that uses a for loop to print the numbers 100, 98, 96,..., 4, 2. (CO3, K2)

Or

(b) Write a program that prints a giant letter A like the one below. Allow the user to specify how large the letter should be (CO3, K3)

```
*
**
***
****
```

19. (a) Write a program that asks the user for their name and how many times to print it. The program should print out the user's name the specified number of times. (CO4, K2)

Or

(b) Explain Type conversion in Python with example. (CO4, K3)

20. (a) Write a program in Python to find the product of the two matrices. (CO5, K4)

Or

(b) Write a program in Python to find the mean, median, mode and standard deviation for the given integers. (CO5, K3)

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R0909

Sub. Code
511401

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   ($10 \times 1 = 10$)

Answer **all** the following objective questions by choosing the correct option.

1. A Banach space is a ___________. (CO1, K1)

   (a) Normed space (b) Complete normed space

   (c) Hilbert space (d) Inner product space

2. The sphere $S\ (0; 1) = \{x \in X \mid \|x\| = 1\}$ in a nomed space X is called the ___________. (CO1, K1)

   (a) Closed sphere (b) Open sphere

   (c) Unit sphere (d) Closed unit sphere

3. An element x of an inner product space X is said to be orthogonal to an element $y \in X$ if ___________. (CO2, K2)

   (a) $\langle x, y \rangle \neq 0$ (b) $\langle y, x \rangle = 0$

   (c) $\langle y, x \rangle \neq 0$ (d) $\langle x, y \rangle = 0$

4. A total set in a normed space X is a subset $M \subset X$ whose span is ___________ in X. (CO2, K1)

(a) Orthogonal (b) Orthonormal

(c) Separable (d) Dense

5. A bounded linear operator $T : H \to H$ on a Hilbert space H is said to be self adjoint if ___________. (CO3, K1)

(a) $T^* = T^{-1}$ (b) $T^*T = T^{-1}T$

(c) $TT^* = TT^{-1}$ (d) $T^* = T$

6. The Hilbert adjoint operator T^* of T is defined by ___________. (CO3, K1)

(a) $\langle Tx, y\rangle = \langle x, T^*y\rangle$

(b) $\langle Tx, y\rangle = \langle x, Ty\rangle$

(c) $\langle Tx, x\rangle = \langle x, T^*y\rangle$

(d) $\langle T^*x, y\rangle = \langle x, T^*y\rangle$

7. A subset M of a metric space X is said to be ___________ in X if its closure $\overline{M}$ has no interior points. (CO4, K1)

(a) Meager (b) Dense

(c) Nowhere dense (d) Nonmeager

8. A vector space X is said to be algebraically reflexive if the canonical mapping is ___________. (CO4, K1)

(a) Injective (b) Bijective

(c) Reflexive (d) Surjective

9. The space $C[a, b]$ is a __________ Banach algebra with identity (CO5, K1)

(a) Associative (b) Commutative

(c) Bounded (d) Complex

10. Let X and Y be normed spaces. A sequence (T_n) of operators $T_n \in B(X, Y)$ is said to be __________ if $(T_n x)$ converges strongly in Y for every $x \in X$. (CO5, K1)

(a) Uniformly operator convergent

(b) Strongly operator convergent

(c) Weakly operator convergent

(d) Convergent

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Show that a compact subset M of a metric space is closed and bounded. (CO1, K4)

Or

(b) If a normed space X is finite dimensional, then show that every linear operator on X is bounded. (CO1, K3)

12. (a) Show that for any subset $M \neq \theta$ of a Hilbert space H, the span of M is dense in H if and only if $M^{\perp} = \{0\}$. (CO2, K3)

Or

(b) Show that the space $C[a, b]$ is not an inner product space, hence not a Hilbert space. (CO2, K3)

13. (a) Let X and Y be inner product spaces and $Q : X \to Y$ a bounded linear operator. Then prove that

(i) $Q = 0$ if and only if $< Qx, y ? = 0$ for all $x \in X$ and $y \in Y$.

(ii) If $Q : X \to X$, where X is complex and $< Qx, x >= 0$ for all $x \in X$, then $Q = 0$.

(CO3, K4)

Or

(b) Prove that the product of two bounded self-adjoint linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute, $ST = TS$. (CO3, K3)

14. (a) Show that every Hilbert space H is reflexive.

(CO4, K3)

Or

(b) Prove that in every Hubert space $H \neq \{0\}$ there exists a total orthonormal set. (CO4, K3)

15. (a) Let (x_n) be a weakly convergent sequence in a normed space X, say, $x_n \overset{w}{\to} x$. Then prove that

(i) The weak limit x of (x_n) is unique.

(ii) Every sub sequence of (x_n) converges weakly to x.

(iii) The sequence $(\|x_n\|)$ is bounded. (CO5, K4)

Or

(b) Let $X = C[0,1]$ and $T : \mathscr{D}(T) \to X$ and $x \mapsto x'$ where the prime denotes differentiation and $\mathscr{D}(T)$ is the subspace of functions $x \in X$ which have a continuous derivative. Then prove that T is not bounded, but is closed. (CO5, K3)

Part C $(5 \times 8 = 40)$

Answer **all** questions not more than 1000 words each.

16. (a) Prove that if Y is a Banach space, then $B(X, Y)$ is a Banach space. (CO1, K5)

Or

(b) Let $X = (X, \|.\|)$ be a normed space. Then show that there is a Banach space $\hat{X}$ and an isometry A from X onto a subspace W of $\hat{X}$ which is dense in $\hat{X}$. Prove that the space $\hat{X}$ is unique, except for isometrics. (CO1, K6)

17. (a) Prove: Two Hubert spaces H and $\widetilde{H}$, both real or both complex, are isomorphic if and only if they have the same Hilbert dimension. (CO2, K5)

Or

(b) Let H be a Hilbert space. Then prove that the followings:

(i) If H is separable, every orthonormal set in H is countable.

(ii) If H contains an orthonormal sequence which is total in H, then H is separable. (CO2, K6)

18. (a) Let H_1, H_2 be Hilbert spaces, $S : H_1 \to H_2$ and $T : H_1 \to H_2$ bounded linear operators and α any scalar. Then prove that

(i) $\langle T^* y, x \rangle = \langle y, Tx \rangle$

(ii) $(S+T)^* = S^* + T^*$

(iii) $(\alpha T)^* = \overline{\alpha} T^*$

(iv) $(T^*)^* = T$

(v) $\|T^* T\| = \|TT^*\| = \|T\|^2$. (CO3, K6)

Or

(b) Let $T : H \to H$ be a bounded linear operator on a Hilbert space H. Then prove that

(i) If T is self-adjoint, $<T\,x,\ x>$ is real for all $x \in H$.

(ii) If H is complex and $< T\,x, x >$ is real for all $x \in H$, the operator T is self-adjoint. (CO3, K5)

19. (a) State and Prove Baire's Category Theorem. (CO4, K5)

Or

(b) Prove that the adjoint operator $T^{\times}$ is linear and bounded and $\|T^{\times}\| = \|T\|$. (CO4, K5)

20. (a) Show that a bounded linear operator T from a Banach space X onto a Banach space Y has the property that the image $T(B_0)$ of the open unit ball $B_0 = B\ (0; 1) \subset X$ contains an open ball about $0 \in Y$. (CO5, K6)

Or

(b) Prove that a sequence (T_n) of operators $T_n \in B\ (X, Y)$ where X and Y are Banach spaces, is strongly operator convergent if and only if

(i) The sequence $(\|T_n\|)$ is bounded.

(ii) The sequence $(T_n x)$ is Cauchy in Y for every x in a total subset M of X. (CO5, K5)

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R0910

Sub. Code
511402

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. $P(A \mid B) =$ (CO1, K1)

   (a) $P(A \cap B)/P(B)$ (b) $P(A \cap B)/P(A)$

   (c) $P(A \cup B)/P(B)$ (d) $P(A \cup B)/P(A)$

2. Suppose X is a real-valued random variable with $E[X] = 4$ and $E[X^2] = 25$. Which of the following statements is true? (CO1, K1)

   (a) $E[X^2] > (E[X])^2$

   (b) $E[X^2] = (E[X])^2$

   (c) $E[X^2] < (E[X])^2$

   (d) It is impossible to determine without more information.

3. ———————— is the subset of sample space. (CO2, K1)

(a) event (b) random scale

(c) outcomes (d) random experiment

4. Identify the incorrect statement regarding the expectation of random variables: (CO2, K1)

(a) $E(X+Y)=E(X)+E(Y)$

(b) $E(aX+Y)=aE(X)+E(Y)$

(c) $E(aX+Y)=E(X)+aE(Y)$

(d) $E(aX)=aE(X)$

5. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by (CO3 K1

(a) $\sqrt{\mu}$ (b) μ^2

(c) μ (d) $\frac{1}{\mu}$

6. Putting α = 1 in Gamma distribution results in (CO3, K1)

(a) Exponential Distribution

(b) Normal Distribution

(c) Poisson Distribution

(d) Binomial Distribution

7. ———————— is the branch of mathematics for collecting, analysing and interpreting data. (CO4, K1)

(a) probability (b) random variable

(c) statics (d) statistic

8. If the between-groups variance and within-groups variance are 250 and 100 respectively and there are 5 groups with 10 observations each, what is the approximate t-value? (CO4, K1)

(a) 2.5 (b) 3.54

(c) 5 (d) 10

9. Which of the following best defines an estimator? (CO5, K1)

(a) A person who hesitates or wavers in making decisions

(b) An approximate calculation of a quantity based on observed data

(c) A rule or formula used to calculate an estimate of a given quantity based on observed data

(d) None of the above

10. Which of the following statements regarding convergence in distribution is true? (CO5, K1)

(a) It guarantees convergence of sample means to population means

(b) It implies convergence of random variables to a fixed value with high probability

(c) It is equivalent to almost sure convergence

(d) It describes the convergence of the cumulative distribution functions of random variables

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Derive the law of total probability. (CO1, K2)

Or

(b) Let X be a random variable with cumulative distribution function $F(x)$. Prove that (CO1, K2)

(i) For all a and B, if $a < b$, then $F(a) \le F(b)$ (F is non-decreasing)

(ii) $\lim_{x \to -\infty} F(x) = 0$

(iii) $\lim_{x \to \infty} F(x) = 1$

(iv) $\lim_{x \downarrow x_0} F(x) = F(x_0)$

12. (a) Let X_1 and X_2 have the joint pdf (CO2, K3)

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & otherwise \end{cases}$$

Find the marginal pdf of X_1 and X_2 Also find $P\left(X_1 \le \frac{1}{2}\right)$ and $P(X_1 + X_2 \le 1)$.

Or

(b) Let (X_1, X_2) have the joint cdf $F(x_1, x_2)$ and let X_1 and X_2 have the marginal cdfs $F_1(x_1)$ and $F_2(x_2)$, respectively. Then show that X_1 and X_2 are independent if and only if $F(x_1, x_2) = F_1(x_1) F_2(x_2)$ for all $(x_1, x_2) \in R^2$.

(CO2, K3)

13. (a) Let X be a random variable such that $E(X^m) = \frac{(m+3)!}{3!} 3^m, m = 1,2,3 \cdots$ Then find the mgf of X. (CO3, K3)

Or

(b) Suppose X has a $\chi^2(r)$ distribution. If $k > -\frac{r}{2}$, show that $E(X^k)$ exists and $E(X^k) = \frac{2^k \Gamma\left(\frac{r}{2} k\right)}{\Gamma\left(\frac{r}{2}\right)}$ if $k > -\frac{r}{2}$.

(CO3, K3)

14. (a) Derive the moments of F-distribution. (CO4, K4)

Or

(b) Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 4 from a distribution having pdf

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Express the pdf of Y_3 in terms of $f(x)$ and $F(x)$ and then compute $P\left(\frac{1}{2} < Y_3\right)$. (CO4, K3)

15. (a) Suppose $X_n \xrightarrow{P} a$ and the real function g is continuous at a. Then show $g(X_n) \xrightarrow{P} g(a)$. (CO5, K3)

Or

(b) Let T_n have a t-distribution with n degrees of freedom, n=1,2,3,... and its cdf is

$$F_n(t) = \int_{-\infty}^{t} \frac{\Gamma[(n+1)/2]}{\sqrt{\pi n}\,\Gamma(n/2)} \frac{1}{(1+y^2/n)^{(n+1)/2}}\, dy,$$

where the integrand is the pdf $f_n(y)$ of T_n. Show that T_n has a limiting standard normal distribution. (CO5, K3)

Part C $(5 \times 8 = 40)$

Answer **all** questions not more than 1000 words each.

16. (a) State and prove Chebychev's inequality. (CO1, K4)

Or

(b) (i) State and Prove Boole's Inequality.

(ii) Show that for any random variable, $P(X = x) = F_X(x) - F_X(x-), \forall x \in R$,where $F_X(x-) = \lim_{z \uparrow x} F_X(z)$. (CO1, K3)

17. (a) State and prove the condition of the equivalence for the independance of two random variables. (CO2, K4)

Or

(b) Let (X_1, X_2) be a random vector such that the variance of X_2 is finite. Then show that

(i) $E[E(X_2 \mid X_1)] = E(X_2)$

(ii) $\mathrm{Var}[E(X_2 \mid X_1) \leq Var(X)]$ (CO2, K3)

18. (a) (i) If the random variable X is $N(\mu, \sigma^2), \sigma^2 > 0$, then show that the random variable $V = \frac{(X-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

(ii) Prove the additive property of the normal distribution under independence. (CO3, K3)

Or

(b) Let $X_1 \cdots X_n$ be independent random variables. Suppose ,for $i = 1, \cdots n$ that X_i has a $\Gamma(\alpha_i, \beta)$ distribution. Let $Y = \sum_{i=1}^{n} X_i$. Then prove that Y has distribution. $\Gamma\left(\sum_{i=1}^{n} \alpha_i, \beta\right)$ distribution. (CO3, K3)

19. (a) Let $Y_1 < Y_2 < \cdots < Y_n$ denote the n order statistics based on the random sample $X_1, X_2 \cdots, X_n$ from a continuous distribution with pdf $f(x)$ and support (a, b). Then derive the joint pdf of $Y_1, Y_2, \cdots Y_n$. (CO4, K3)

Or

(b) (i) If the random variable X has a poisson distribution such that $P(X=1) = P(X=2)$,find $P(X=4)$.

(ii) Derive t-distribution (CO4, K3)

20. (a) State and prove the weak law of large numbers. (CO5, K3)

Or

(b) State and prove central limit theorem. (CO5, K4)

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R0911

Sub. Code
511403

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. A ———— of G is a complete subgraph of G (CO1, K2)
   (a) Clique (b) Complete
   (c) Connected (d) subgraph

2. In any Graph G, the number of vertices of ———— is even. (CO1, K2)
   (a) Even degree (b) Odd degree
   (c) Both (a) and (b) (d) None of these

3. Each block of G with atleast three vertices is a subgraph of G. (CO2, K2)
   (a) Unconnected (b) Connected
   (c) 1-connected (d) 2-connected

4. The degree of each vertex of G is an ———— integer. (CO2, K2)
   (a) Positive (b) Negative
   (c) Even positive (d) Odd positive

5. A matching in G is a set of ―――――― edges. (CO3, K4)

(a) Dependent (b) Independent

(c) Vertex edges (d) Perfect

6. A regular bipartite graph is ―――――― (CO3, K4)

(a) 2-fabtorable (b) 3-fabtorable

(c) 4-fabtorable (d) None of these

7. If a connected graph G is neither an odd cycle nor a complete graph, then ―――――― (CO4, K2)

(a) $\chi(G) \le \Delta(G)$ (b) $\chi(G) \ge \Delta(G)$

(c) $\chi(G) = \Delta(G)$ (d) $\chi(G) = \Delta(G) = 0$

8. The ―――――― $a(G)$ of a graph G is the maximum k for which G has a complete k coloring. (CO4, K2)

(a) Edge chromatic number

(b) achromatic number

(c) pseudochromatic number

(d) both (a) and (b)

9. A graph is ―――――― if and only of it is embeddable on a sphere. (CO5, K6)

(a) Plane (b) Planar

(c) Embeddable (d) Sphere

10. If G is a simple planar graph with atleast the vertices, then ―――――― (CO5, K6)

(a) $m \ge 3n - 6$ (b) $m = 3n - 6$

(c) $m \le 3n - 6$ (d) $m < 3n - 6$

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Show that sum of the degrees of the vertices of a graphs is equal to twice the number of its edge. (CO1, K2)

Or

(b) Explain isomorphism of graphs. (CO1, K2).

12. (a) Explain the Block. (CO2, K2)

Or

(b) Show that *G* is Hamiltonian, then for every nonempty proper subset *S* of *V*. (CO2, K2)

13. (a) Explain the Hall theorem. (CO3, K2)

Or

(b) Explain the Konig theorem (CO3, K4)

14. (a) Compare the chromatic number and edge chromatic number. (CO4, K4)

Or

(b) Solve the chromatic polynomials. (CO4, K2)

15. (a) Elaborate the Dual graphs. (CO5, K6)

Or

(b) Discuss the plane and planar graphs. (CO5, K6)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1,000 words each.

16. (a) Show that $\tau(k_n) = n^{n-2}$ where k_n is a labeled complete graph on n vertices, $n \geq 2$. (CO1, K2)

Or

(b) Compare the cut edge and Bonds. (CO1, K2)

17. (a) Show that a connected simple graph *G* is 3-edge connected if and only if every edge of *G* is the intersection of the edge sets of two cycles of *G*. (CO2, K2)

Or

(b) Examine the Ear decomposition of a block. (CO2, K4)

18. (a) Examine the Ramsey's theorem. (CO3, K4)

Or

(b) Compare the matching and coverings in bipartite graphs. (CO3, K2)

19. (a) Show the Vizing's theorem. (CO4, K2)

Or

(b) Explain the Brook's theorem. (CO4, K2)

20. (a) Formulate the Eulers formula. (C05, K6)

Or

(b) Discuss the five colour theorem and Four colour conjecture. (CO5, K6)

R0912

Sub. Code
511404

M.Sc. DEGREE EXAMINATION, APRIL – 2024

Fourth Semester

Mathematics

MEASURE AND INTEGRATION

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 1 = 10$)

Answer **all** the following objective type questions by choosing the correct option.

1. The Lebesgue outer measure of the set $\{x \in \mathbb{R} \mid 0 < x^2 < 2\}$ is ____________ (CO1, K2)

   (a) 0 (b) $\sqrt{2}$

   (c) $2\sqrt{2}$ (d) 2

2. Let $f : [0,1] \to \mathbb{R}$ be the function defined by (CO1, K2)

   $$f(x) = \begin{cases} x, & if\ x \in [0,1] \cap \mathbb{Q} \\ x^2, & if\ x \in [0,1] \cap \mathbb{Q}^c \end{cases}$$

   Then pick out the true statement.

   (a) f is measurable and $f = g$ a.e., where $g(x) = x$ for all $x \in [0,1]$

   (b) f is measurable and $f = h$ a.e., where $h(x) = x^2$ for all $x \in [0,1]$

   (c) $f = g$ a.e., where $g(x) = x$ for all $x \in [0,1]$ but not measurable

   (d) $f = h$ a.e., where $h(x) = x^2$ for all $x \in [0,1]$ but not measurable

3. Which of the following is NOT necessarily true? (CO2, K2)

(a) There exist no non-negative measurable function f such that $\int_{[0,1]} f dx = 0$

(b) There exist no non-positive measurable function f such that $\int_{[0,1]} f dx = 0$

(c) There exist no monotonically increasing continuous function f such that $\int_{[0,1]} f dx = 0$

(d) There exist a non-zero measurable function such that $\int_{[0,1]} f dx = 0$

4. Let $f : [0,1] \to \mathbb{R}$ be the function defined by $f(x) = 0$ for x rational, if x is irrational, $f(x) = n$, where n is the number of zeros immediately after the decimal point, in the representation of x on the decimal scale. Then $\int_0^1 f dx =$ ____________ (CO2, K2)

(a) 1 (b) 0

(c) $\frac{1}{10}$ (d) $\frac{1}{9}$

5. Let $f : (a, b) \to \mathbb{R}$ and $x \in (a, b)$. Then with respect to standard notations which of the following is NOT true? (CO3, K4)

(a) $D^+(-f(x)) = -D^+ f(x)$

(b) $D^+(-f(x)) = -D_+ F(x)$

(c) $D^+ f(x) \geq D_+ f(x)$

(d) $D^- f(x) \geq D_- f(x)$

6. The Lebesgue set a monotone increasing function on $[0,1]$ is always ———— (CO3, K4)

(a) Countable (b) Uncountable

(c) Empty (d) Finite set

7. Which of the following is NOT necessarily true? (CO4, K2)

(a) Countable union of positive sets is a positive set

(b) Countable union of negative sets is a negative set

(c) Countable union of null sets is a null set

(d) Countable union of null sets is need not to be a null set

8. Let $[X, S, \mu]$ be a finite measure space. Then pick out the correct statement (CO4, K2)

(a) For each non-negative measurable function f, there is no measure v on S such that v is absolutely continuous with respect to μ

(b) For each non-negative measurable function f, the set function v defined by $v(E)=\int_E f d\mu$ for all $E \in S$ is a measure on S such that μ is absolutely continuous with respect to v

(c) For every measure v on S which is absolutely continuous with respect to μ there is no non-negative measurable function f such that $v(E)=\int_E f d\mu$ for all $E \in S$

(d) For every measure v on S which is absolutely continuous with respect to μ there exists a non-negative measurable function f such that $v(E)=\int_E f d\mu$ for all $E \in S$.

9. A class $\mathcal{M}_0$ of subsets of a space is a monotone class if for any increasing or decreasing sequence of sets $\{E_n\}$ of $\mathcal{M}_0$ then ———— (CO5, K4)

(a) $\lim_{n\to\infty} E_n \in \mathcal{M}_0$ (b) $\lim_{n\to\infty} E_n \notin \mathcal{M}_0$

(c) $\lim_{n\to\infty} E_n^c \in \mathcal{M}_0$ (d) $\lim_{n\to\infty} E_n^c \notin \mathcal{M}_0$

10. Let $\mathscr{M}$ denote the collection of all Lebesgue measurable sets in $\mathbb{R}$ and m be the Lebesgue measure on $\mathscr{M}$. Then the measure of the set $\{(x, y) | x, y \in \mathbb{Q}^c,\ 1 \le x \le \sqrt{2}, 0 \le y \le 1\}$ with respect to the product measure $m \times m$ is ———— (CO5, K4)

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$

(c) $\sqrt{2}$ (d) $2\sqrt{2} - 1$

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Let E be any set of real numbers. Then prove that the following are equivalent: (CO1, K2)

(i) E is measurable.

(ii) For each $\in > 0$, there is an open set O such that $E \subseteq O$ and $m*(O \sim E) < \varepsilon$

(iii) There is a G_δ- set G such that $E \subseteq E$ and $m*(G \sim E) = 0$.

Or

(b) If $\mu*$ is the outer measure on $\mathscr{H}(\mathscr{R})$ induced by the measure μ on $\mathscr{R}$, then prove that $S(\mathscr{R})$ is contained in $S*$. (CO1, K2)

12. (a) State Dominated Convergence Theorem and sketch a proof of it. (CO2, K2)

Or

(b) Show that $\lim_{n\to\infty}\int_0^\infty \frac{dx}{(1+x/n)^n x^{1/n}} = 1$. (CO2, K2)

13. (a) If $f \in L(a,b)$ then prove the following : (CO3, K4)

(i) $F(x) = \int_a^x f(t)\,dt$ is a continuous function on $[a,b]$

(ii) $F \in BV[a,b]$.

Or

(b) If $f \in L(a,b)$ and $\int_a^x f\,dt = 0$ for all $x \in (a,b)$ then prove that $f = 0$ a.e. in (a,b). (CO3, K4)

14. (a) Let v be a signed measure on $[X,S]$. Then prove that there exists a positive set A and a negative set B such that $X = A \cup B$ with $A \cap B = \phi$. (CO4, K3)

Or

(b) Let v be a signed measure on $[X,S]$. Let $E \in S$ and $v(E) > 0$. Then prove that there exists a positive set A with respect to v such that $A \subseteq E$ and $v(A) > 0$. (CO4, K3)

15. (a) Let f be an $\mathcal{S} \times \mathcal{J}$- measurable function. Then prove that for each $x \in X$ and $y \in Y$, f_x is a J-measurable function and f^y is an S-measurable function. (CO5, K4)

Or

(b) Let f be non-negative $\mathcal{S} \times \mathcal{J}$-measurable function and let $\phi(x)=\int_Y f_x\, dv, \psi(y)=\int_X f^y\, d\mu$ for each $x \in X$, $y \in Y$. Then prove that ϕ is S-measurable, ψ is J-measurable and

$$\int_X \phi\, d\mu = \int_{X \times Y} fd(\mu \times v) = \int \psi\, dv .$$ (CO5, K4)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) Prove that there exists non-measurable set. (CO1, K2)

Or

(b) Let μ^* be an outer measure on $\mathcal{H}(\mathfrak{R})$. Then formulate a proof that the set of all μ^* measurable sets S^* is a σ-ring. Also, prove that $\mu^* | S^*$ is a complete measure. (CO1, K2)

17. (a) State Fatou's Lemma and formulate a proof of it. (CO2, K2)

Or

(b) Let f and g be two integrable functions. Then show that the following results are true. (CO2, K2)

(i) αf is integrable and $\int \alpha f\, dx = \alpha \int f dx$ for all $\alpha \in \mathbb{R}$.

(ii) $f + g$ is integrable and $\int (f + g)\, dx = \int f\, dx + \int g\, dx$.

(iii) If $f = 0$ a.e., then $\int f dx = 0$

(iv) If $f \leq g$ a.e., then $\int f dx \leq \int g dx$.

18. (a) Let $[a, b]$ be a finite interval and let $f \in L(a, b)$ with indefinite integral F. Then prove that $F' = f$ a.e. in $[a, b]$. (CO3, K4)

Or

(b) If $f \in L(a,b)$ where (a, b) is a finite interval, then show that there exists a set $E \subseteq (a, b)$ such that $m^*([a,b] \sim E) = 0$ and

$$\lim_{h \to 0} \frac{1}{h} \int_x^{x+h} |f(t) - \xi| \, dt = |f(t) - \xi|$$ for all real ξ and all $x \in E$. (CO3, K4)

19. (a) State and prove Jordon decomposition theorem. (CO4, K3)

Or

(b) Let $[X, S, \mu]$ be a finite measure space and ν be finite measure on S such that $\nu << \mu$. Then prove that there exists a finite valued non-negative measurable function f on X such that $\nu(E) = \int_E f d\mu$ for all $E \in S$. (CO4, K3)

20. (a) State Fubini's theorem and formulate a proof of it. (CO5, K4)

Or

(b) Let $[X, S, \mu]$ and $[Y, J, \nu]$ be finite measure space. For $V \in S \times J$ define $\phi(x) = \nu(V_x)$, $\psi(y) = \nu(V^y)$ for all $x \in X$, $y \in Y$. Then prove that ϕ is S-measurable, ψ is J-measurable and $\int_X \phi \, d\mu = \int_Y \psi \, d\nu$. (CO5, K4)

———————